Self-Reciprocity Calibration of Electroacoustic Transducers

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Self-Reciprocity Calibration of Electroacoustic Transducers*

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By an extension of the reciprocity principle for the absolute measurement of sound, it is demonstrated that a calibration may be obtained on a single transducer without the aid of auxiliary transducers. Measurements using the technique are described and shown to give satisfactory agreement with conventional calibrations.

IT has been shown[1] that the receiving and transmitting responses of linear, passive, reversible electroacoustic transducers are related by a parameter which is independent of the geometry or construction of the transducer. This parameter can easily be computed from the following relationship:

$$\frac{M_0}{S_0} = J = \frac{2D\lambda}{R} \cdot 10^{-7},$$

where:

J = reciprocity parameter

M_0 = the microphone response in open circuit volts per dyne per cm^2

S_0 = the speaker response in dynes per cm^2 at a distance D centimeters per ampere input current

λ = wave-length in centimeters

R = characteristic acoustic resistance of the medium in g/cm^3/sec.

Using a method suggested by MacLean,[1] The Underwater Sound Reference Laboratories, during the recent war, pioneered in the application of this basic principle to the absolute calibration of standard transducers. Calibrations made by this method were shown to agree satisfactorily with methods previously recognized as valid in the acoustic field.

For a single absolute measurement three sets of voltage measurements and three transducers are required: a projector, a reversible transducer, and a hydrophone.[2] Thus although the technique is at present widely used for absolute calibrations, because of its complexity, it is ordinarily employed only to obtain calibrations on primary standards which can in turn be used for comparison calibrations.

This paper discusses an application of the reciprocity relationship, by which it is possible to make absolute measurements of sound pressures using a single transducer in the sound field of its image with a single set of voltage measurements. From the nature of the calibration this method has been called "Self-Reciprocity."

I. METHOD

The transducer is located relative to a reflector as shown in Fig. 1. A short pulse of sinusoidal current I_s is applied to the transducer, which generates a corresponding pulse of acoustic energy. This pulse strikes a perfect reflector at a distance d centimeters from the transducer face and upon returning excites in the transducer an open circuit voltage E_m.

By using the following definitions of hydrophone and projector response

$M_0 = E_m/p_m$, where p_m is the free-field acoustic pressure in the reflected signal at the face of the transducer,

and

$S_0 = P_1/I_s$, where P_1 is the acoustic pressure at 100 centimeters from the projector diaphragm,

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[1] Walter Schottkey, "Das Gesetz des Tiefempfangs in der Akustik und Electroakustik" Zeits. f. Physik 36, 689–736 (1926); W. R. MacLean "Absolute measurement of sound without a primary standard," J. Acous. Soc. Am. 12, 140–146 (1940); L. L. Foldy and H. Primakoff "A general theory of passive linear electroacoustic transducers and the electroacoustic reciprocity theorem," J. Acous. Soc. Am. 17, 109–120 (1945) and 19, 50–53 (1947).

[2] The measurements under discussion were made in the field of underwater sound. The method, however, is general, and for application to air acoustics the words loudspeaker and microphone may be substituted for projector and hydrophone, respectively.
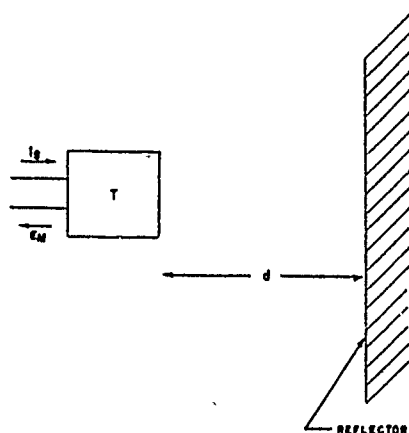
Fig. 1.

in the reciprocity relationship,

$$\frac{M_0}{S_0} = J, \qquad (1\text{-}1)$$

we find

$$\frac{E_m I_s}{p_m P_1} = J. \qquad (1\text{-}2)$$

If we choose a distance d in Fig. 1 large in comparison with the dimensions of the transducer, we can assume a spherical wave and obtain a relationship between the pressures p_m and P_1

$$\frac{p_m}{P_1} = \frac{100}{2d}. \qquad (1\text{-}3)$$

Combining (1-1), (1-2), and (1-3) we have

$$M_0 = \left(\frac{E_m}{I_s} \frac{d}{50} J\right)^{\frac{1}{2}}, \qquad (1\text{-}4)$$

and

$$S_0 = \left(\frac{E_m}{I_s} \frac{d}{50J}\right)^{\frac{1}{2}}. \qquad (1\text{-}5)$$

II. EXPERIMENTAL SET-UP

Figure 2 shows the arrangement of equipment used in the measurements. A millisecond pulse of electrical energy from the driver is supplied to the transducer through a coupling transformer. A pulse repetition rate is selected which is sufficiently low that all reflections and reverberations will have been dissipated at the time of arrival of the following pulse. The voltage caused by the reflected signal is measured by a high input impedance receiving system. The amplified signal is then displayed on a cathode-ray oscilloscope screen. The open circuit voltage E_m can be determined from a calibration of the receiving system which includes the coupling loss of the transducer circuit. This is obtained by observing the C.R.O. deflection when a known voltage is generated in series with the transducer. The current is measured by determining the voltage across a small known resistor inserted in series with the transducer. It is possible for both the receiving system calibration and the current measurement to be made on continuous wave.

The sensitivity of the measuring system is a function of the impedance ratio of the coupling transformer mentioned above. Assuming a constant output impedance from the driver, it is shown below that the maximum over-all sensitivity of the system is obtained when the output impedance of the transformer R_0 is three times that of the transducer. Figure 3 shows that the driver R_0 becomes the load for the transducer R_t
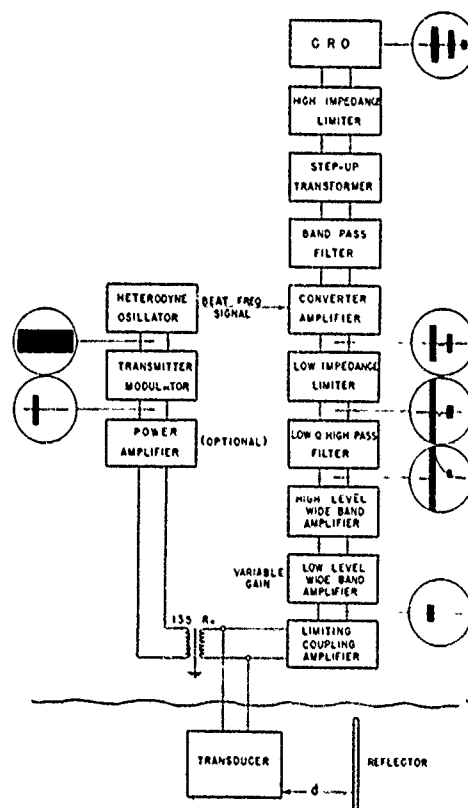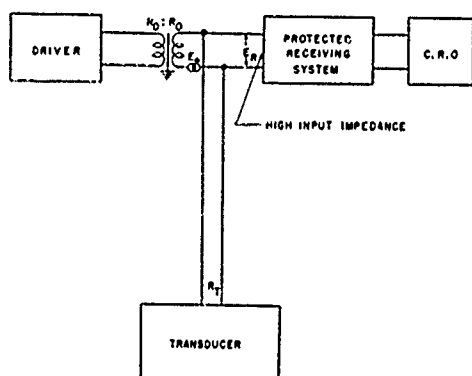

Fig. 2. Block diagram of equipment.

FIG. 3. Coupling between driver and transducer.

on receiving. Thus the voltage E_R measured by a high input impedance receiving system is:

$$E_R = \frac{E_m R_0}{R_0 + R_t}. \tag{2-1}$$

From Eq. (1-3)

$$E_m = \frac{M_0^2}{2dJ} I_s, \tag{2-2}$$

and

$$I_s = \frac{E_0}{R_0 + R_t}.$$

For simplicity assume all impedance to be resistive.

The output impedance R_0 of the coupling transformer can be varied by changing the ratio of R_0 to R_d, the constant output impedance of the driver. Thus E_0, the equivalent generator voltage at the output of the transformer, may be expressed by

$$E_0 = k R_0^{\frac{1}{2}}$$

and

$$I_s = \frac{k R_0^{\frac{1}{2}}}{R_0 + R_t}. \tag{2-3}$$

Combining (2-1), (2-2), and (2-3) and lumping constant terms,

$$E_R = K \frac{R_0^{\frac{3}{2}}}{R_0 + R_t}. \tag{2-4}$$

To determine the value of R_0 for which E_r is a maximum, the first derivative with respect to R_0 of Eq. (2-4) is set equal to zero, resulting in

$$0 = \frac{3}{2} - \frac{2R_0}{(R_0 + R_t)}.$$

Thus for E_r a maximum

$$R_0 = 3R_t. \tag{2-5}$$

In certain of the measurements the output stage of the driver was keyed directly. In this case, the output impedance of the driver changed from a low impedance while driving to a relatively high impedance when the driver tubes were cut off. This was a particularly valuable feature in these measurements because the output impedance could be matched to the transducer to give high current during the transmitting pulse, but would then, on cut-off, change to a high impedance and thus give a smaller coupling loss for the received signal.

The primary practical problem was to obtain an electronic measuring system which would be sensitive enough to measure accurately the voltage E_m and yet remain unaffected by the application of the large driving voltage. These voltages may differ in magnitude by more than 100 db for many transducers. In the work carried out at this laboratory, the most direct and effective method for protecting the receiving system from the shock of the transmitting signal was found to be in limiting the received voltage so that no element of the system could be overloaded.

This was accomplished by inserting diode limiters (see Fig. 4) before the inputs to certain active elements. A flexible and stable system was achieved by partial limiting at strategic locations between stages of gain such that no grid received a voltage greater than its negative bias. These limiters were used in various ways to increase their limiting effectiveness. Because of the internal resistance of the diode, it was necessary that they be used in high impedance circuits.

Even with limiting, which completely prevented overloading, i.e., driving grids positive, there were low frequency transients caused by the shift in operating point of various tubes along
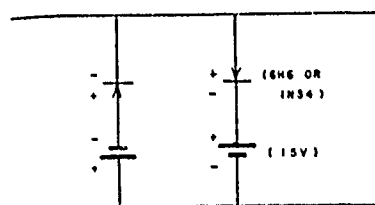

FIG. 4. Typical diode limiter.

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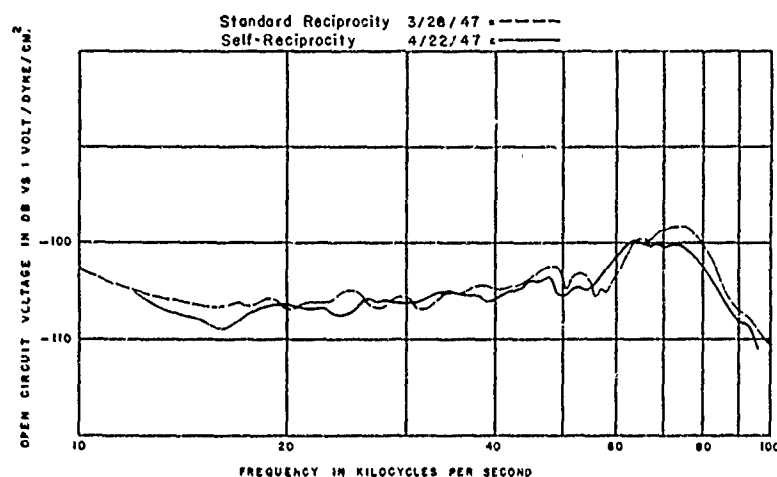Self-Reciprocity   4/22/47 •———

FIG. 5. Receiving response,
6B-35 transducer.

the amplification path. These low frequency components were filtered out. The Q of the filters used, however, was sufficiently low that the shape of the pulse was unaffected. Figure 2 shows the pulse form as observed at various points in the circuit which amplified, limited, and filtered the received signals. Thus in the final signal presented on the C.R.O. there appears a limited transmitted pulse followed by the amplified reflection pulse which in turn is followed by other multiple reflections.

The minimum measurable voltage for the system was of the order of magnitude of −90 db vs. 1 volt.

III. ACOUSTIC MEASUREMENTS

The acoustic measurements were made using the open water calibration facilities of the

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FIG 6. Receiving response, CAEK-102 transducer.

Underwater Sound Reference Laboratory. The transducer was located at a depth of 250 centimeters at a distance of 2 meters from a vertical reflector. The reflector consisted of a $4' \times 4' \times \frac{1}{4}''$ air baffle. It has been shown by theory and experiment that the reflection coefficient of this baffle can be considered unity for the purposes of these tests.

So long as the baffle is large, the depth of the transducer is not critical. From the geometry of the physical arrangement we see also that the orientation of the baffle is not critical so long as it is possible to draw a normal to it from the transducer. The angle of maximum response of the transducer can be selected by rotating the transducer physically and observing the reflected pulse on the C.R.O. Note here that the directional characteristic of the device is doubled in this orientation procedure.

Measurements were made on three typical transducers, which are used by the Laboratory as standard sound sources and hydrophones: BTL Types 6B and 5A crystal, piston sources, and a CAEK 51095 magnetostriction line source.

The results of the self-reciprocity calibrations are shown in Figs. 5–7 along with calibrations made by conventional comparison or reciprocity techniques. The agreement between self-reciprocity calibrations and conventional calibrations is within experimental error.

IV. LIMITATIONS AND APPLICATIONS

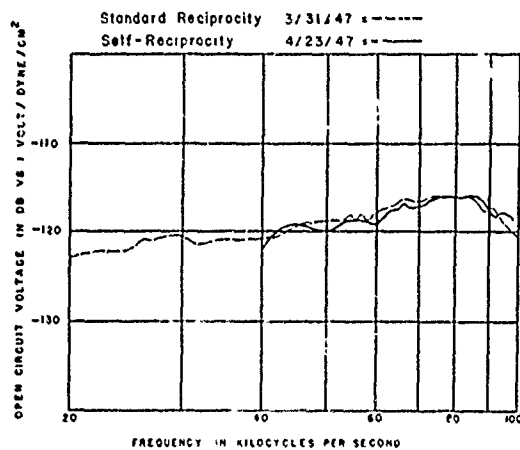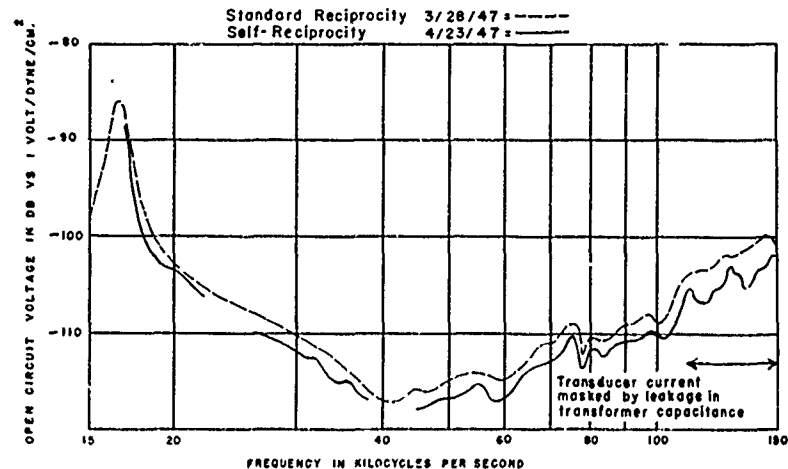The self-reciprocity method of calibration carries with it all the limitations of pulsing. The

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FIG. 7. Receiving response, 5A-8 transducer.

Transducer current masked by leakage in transformer capacitance

FREQUENCY IN KILOCYCLES PER SECOND

pulse length must be sufficiently short that the received pulse can be separated in time from the transmitted pulse or echos from undesirable reflecting surfaces. Yet the pulse must contain a few cycles of the signal frequency. Thus measurements using conveniently short separation between transducer and baffle are limited to frequencies above approximately 5 kilocycles per second. The method also carries with it all the advantages of pulsing in calibration work: i.e., elimination of the effect of cross-talk, standing waves, and other reflections.

The method will be particularly valuable in testing under hydrostatic pressure in tanks. With present methods, because of the rigging difficulties associated with standard reciprocity, it is highly impractical to determine the absolute behavior of a single transducer with varying hydrostatic pressures. Since two transducers are always involved, only the combined response of the two transducers can be measured as the pressure changes. However, in self-reciprocity, since only a single transducer is involved, absolute measurements can be made on that transducer under pressure as simply as in the free field.

One of the principal problems in small tanks is the limitation resulting from the side reflections. The separation required between transducer and baffle in self-reciprocity for a given spherical wave correction is ordinarily one-half that required between two transducers. Consequently from the standpoint of the time margin between direct and

reflected signal, the tank size is effectively greater with the self-reciprocity technique than by standard two-transducer techniques.

In free-field calibrations, self-reciprocity will be of advantage primarily because of its simplicity. The time required for absolute measurements by self-reciprocity compares favorably with that required for a comparison calibration.

Another possibility for the self-reciprocity technique presents itself upon inspection of Eq. (1-4). It will be observed in this equation that d and J are constants. It would presumably be possible to develop a recording-receiving system which would have uniformly constant gain at all frequencies and also to develop a driver which would supply a constant current regardless of transducer impedance. Under these conditions the recording of the response E_m would be directly proportional to the square of the receiving sensitivity of a transducer. Thus it would be fundamentally very simple to devise a system which would give a direct plot of transducer response eliminating the need for processing of the data.

ACKNOWLEDGMENT